Pion-Nucleon Phase Shifts in Lattice QCD

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For details see: C.B. Lang, VV Phys. Rev. D 87, 054502 (2013), arXiv:1212.5055

Outline

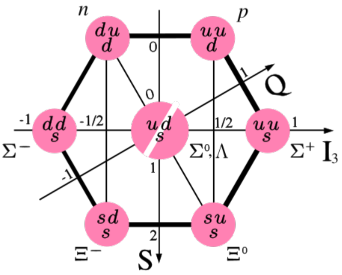
- **Introduction.** Motivation and techniques used in baryon spectroscopy on the lattice.
- **Pion-Nucleon on the lattice.** The nucleon spectrum is not well reproduced by lattice calculation and needs a deeper investigation.
- **Results.** The study of multi-particle systems drastically changes the observed scenario.

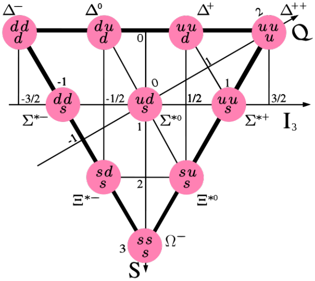
Lattice QCD

- Lattice QCD is formulated on a discrete Euclidean space-time grid that acts as a non-perturbative regularization scheme.
- The only input required are the strong coupling constant and the bare masses of the quarks.
- Lattice QCD provides a non-perturbative tool for calculating the hadronic spectrum from first principles.

Baryon spectroscopy on the lattice

Reproducing the baryon masses starting from the basics degrees of freedoms represents a strong test of the correctness of QCD.

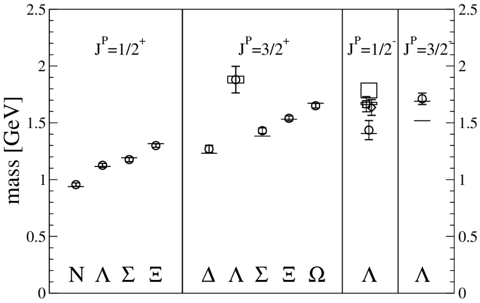




All prediction of LQCD have to match with experimental data!

[Pictures from Wikipedia]

Baryon ground states on the lattice

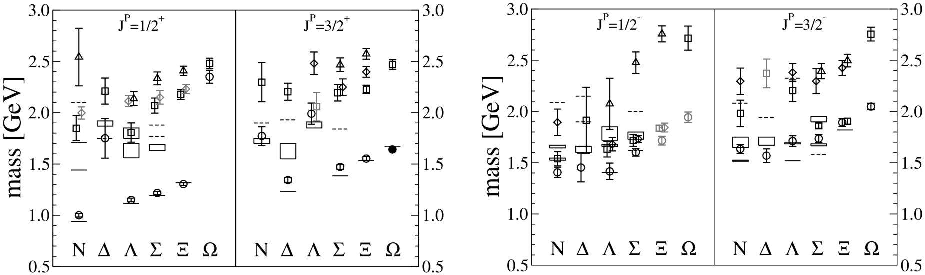


Lattice QCD successfully estimates the ground states of the baryon spectrum

but...

[Engel et al. Phys. Rev. D 87, 074504 (2013)]

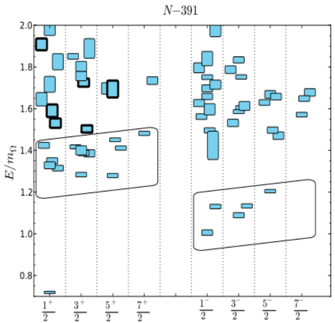
Excited states on the lattice

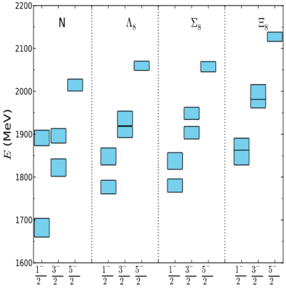


[Engel et al. Phys. Rev. D 87, 074504 (2013)]

Excited states still represent an outstanding challenge.

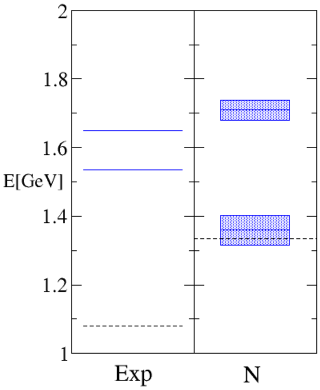
Excited states on the lattice





[Edwards et al. Phys. Rev. D 87, 054506 (2013)]

N^- spectrum



Lattice simulations have problems in reproducing the negative parity sector of the nucleon spectrum.

Why?

These states are unstable under strong interactions and their resonant nature should be taken into account.

N^- decay channels

$N(1535) \to N\pi$	35-55%
$N(1535) \to N\eta$	32-52%

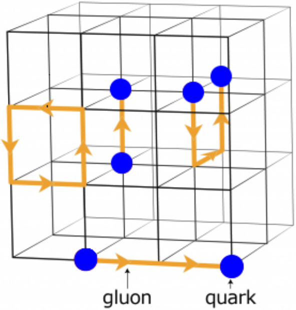
$N(1650) \to N\pi$	50-90%
$N(1650) \to N\eta$	5-15%
$N(1650) \to \Lambda K$	3-11%

$N\,\pi$ is the main decay channel of N^*

$$N^- \longleftrightarrow N\pi \qquad \mathbf{S-wave}$$

Mass spectroscopy on the lattice: ingredients

- Define quark and gluon d.o.f on the lattice and construct the action $S = S_G[U] + S_F[\bar{\psi}, \psi]$
- Use Monte Carlo techniques to produce gauge configurations with Boltzmann distribution e^{-S}.
- Measure the appropriate observable in order to estimate the masses of the QCD spectrum: the hadron correlator function.

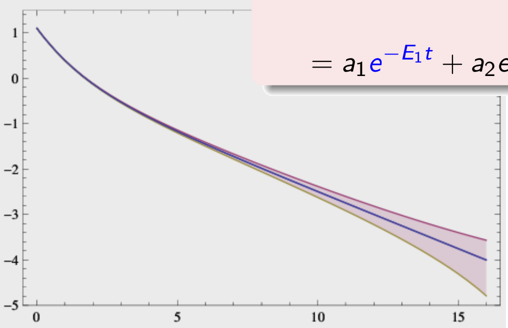


Hadron correlation function

The hadron correlation function in the Euclidean is defined as

$$C_{ij}(t) = \langle \chi_i(t) \chi_j^\dagger(0) \rangle = \sum_n \langle 0|\chi_i|n\rangle e^{-E_n t} \langle n|\chi_j^\dagger|0\rangle =$$

$$= a_1 e^{-E_1 t} + a_2 e^{-E_2 t} + a_3 e^{-E_3 t} + \ldots + \text{noise}$$



$$m_i^2 = E_i^2 - \mathbf{p}^2$$

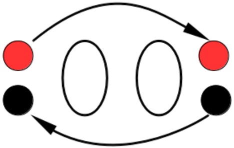
Compute the correlation function: mesons

The correlator involves terms like

$$C(x,0) = \langle \pi(x)\pi(0)^\dagger \rangle = \langle\, D\ \bar{q}(x)\, \underbrace{q(x)\ \bar{q}(0)}_{M^{-1}(x,0)}\, q(0)\, \rangle .$$

It is required to solve $N_S^3 \times N_T \times N_C$ equations like

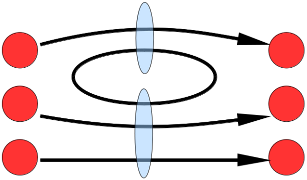
$$M(x,0)\phi(0) = \eta(x) \qquad \rightarrow \qquad \phi(x) = M^{-1}(x,0)\eta(0)$$



Compute the correlation function: baryons

$$C(x,0) = \langle N(x)\bar{N}(0)\rangle = \langle\, D\;\, q_a(x)\, q_b(x)\, \underbrace{q_c(x)\; \bar{q}_e(0)}_{M^{-1}(x,0)}\, \bar{q}_f(0)\, \bar{q}_g(0)\rangle.$$

$N, N', \ldots$



$N, N', \ldots$

The coupling to intermediate states seems not to be strong enough.

$N\pi$ scattering: the issues

- **Many diagrams**: large amount of cpu time needed.
- **Backtracking quark lines**: not affordable with traditional techniques.
- **Many energy levels**: how to extract them reliably?
- **Resonances**: how to treat them?

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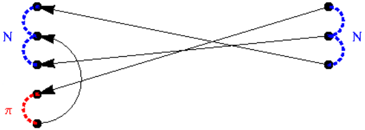
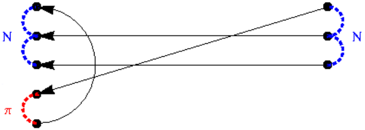
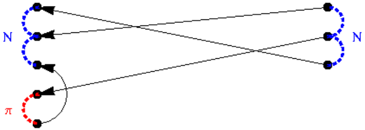
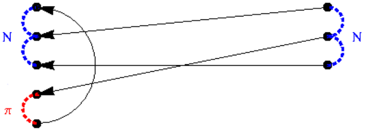
$N\pi \rightarrow N$

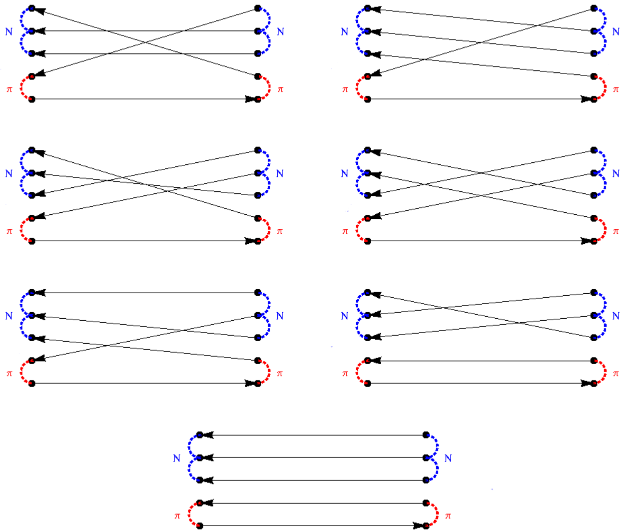




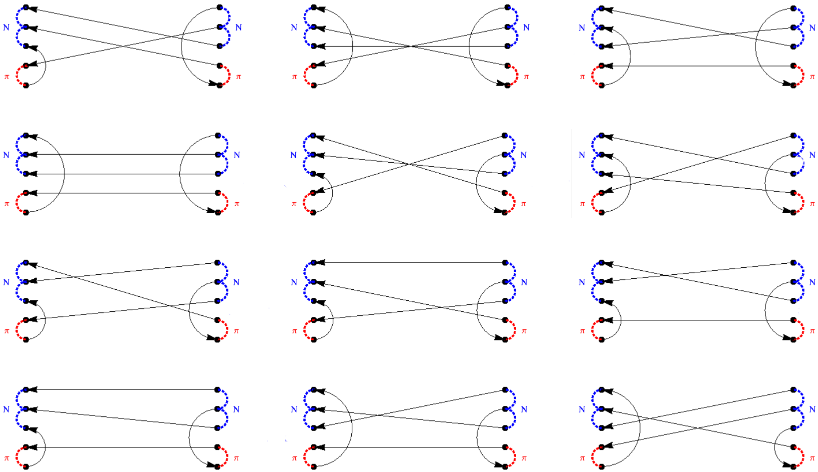




$N\pi \rightarrow N\pi$: connected diagrams



$N\pi \rightarrow N\pi$: partially disconnected diagrams



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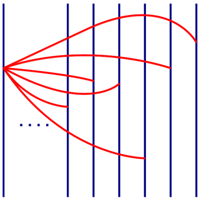
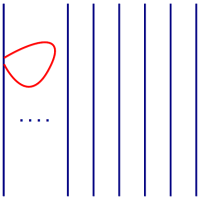
Quark propagator

- Point-to-all method: compute the propagator for one localized source at given time slice to all the lattice. It works for correlators of single hadron operators concerning connected diagrams.





To evaluate backtracking loops we need to consider many sources on each time slice: all-to-all propagator.

Distillation Method [Peardon et al, Phys. Rev. D 80 (2009) 054506]

Smeared sources + Cut measurement costs

Smearing the quarks with a very low rank operator written in terms of eigenvectors of the 3D Laplacian

$$q(x) \longmapsto S(x,x')q(x') = \sum_{i=1}^{N_V} v_i(x)v_i^\dagger(x')q(x')$$

Distillation

After the distillation

$$C = \sum_{\ldots} \sum_{i,j} \underbrace{\Gamma \ldots v_i}_{\Phi} \underbrace{\boxed{v_i^\dagger \, q \, \bar{q} \, v_j}}_{\tau(i,j)} \underbrace{v_j^\dagger \ldots \Gamma}_{\Phi^\dagger}$$

$$\tau_{ij} = v_i^\dagger(x) M^{-1} v_j(y)$$

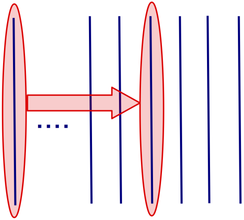
instead of

$$M^{-1}(x, y)$$

$N_V (N_T N_d)$ inversions

$N_S^3 N_C (N_T N_d)$ inversions

Distillation



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Extract the excited states: variational method

[Michael, NPB 259 (1985) 58] [Luescher, Wolff. NPB 339 (1990) 222]

Consists in disentangling the states using several interpolators.

- Use several interpolators χ_i to construct a basis with minimum overlap.
- Compute the cross correlations $C_{ij}(t) = \langle \chi_i(t)\chi_j^\dagger(0)\rangle$
- Solve the generalized eigenvalue problem

$$C(t)u^{(n)} = \lambda^{(n)}C(t_0)u^{(n)}$$

- Obtain energy levels from the eigenvalues

$$\lim_{t\to\infty} \lambda^{(n)}(t, t_0) = e^{-E_n(t-t_0)}$$

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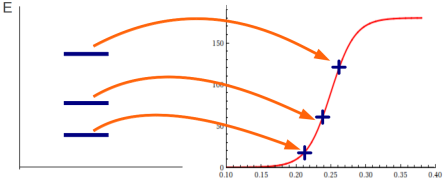
Phase shift analysis [Luescher, Commun. Math. Phys. 104, 177 (1986)]

Asymptotically only stable states can be observed and resonances have to be identified by their impact on the finite volume states.



Luescher formula connects the discrete spectrum in finite volume with the elastic scattering phase shift in infinite volume

$$\det[e^{2i\delta}(M(q) - i) - (M(q) + i)] = 0$$

Simulation setting

- Wilson Clover action with 2 degenerate flavours.
- Configurations: 280
- Lattice size: $16^3 \times 32 \;\; (a = 0.12\,\mathrm{fm})$
- Pion masses: 266 MeV

[A. Hasenfratz et al., Phys. Rev. D 78 (2008) 054511]

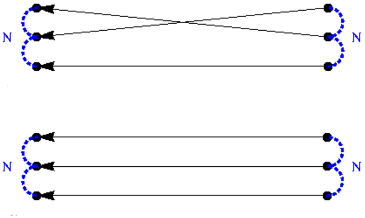
$N \to N$

A good interpolator for the nucleon has the form

$$\chi_i(\mathbf{0}) = \sum_x P_\pm \epsilon_{abc}\, \Gamma_1^i\, u_a(x)\, [u_b^T(x)\Gamma_2^i\, d_c(x) - d_b^T(x)\Gamma_2^i\, u_c(x)]$$

- χ_1 : $(\mathbf{1},\ C\gamma_5)$
- χ_2 : $(\gamma_5,\ C)$
- χ_3 : $(i\mathbf{1},\ C\gamma_4\gamma_5)$

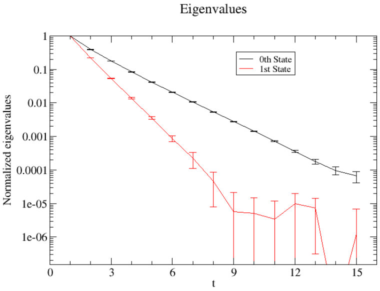
- $P_\pm = (1 \pm \gamma_0)/2$

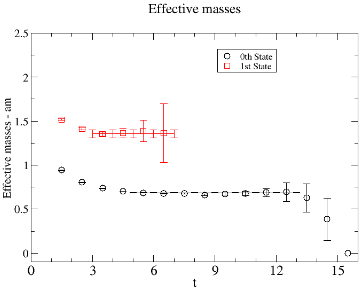


The positive parity sector

$N \to N : N^+$

280 configs, 6 interpolators: $32, 64$ source and sink eigenvectors





C.B. Lang, VV Phys. Rev. D 87, 054502 (2013), arXiv:1212.5055

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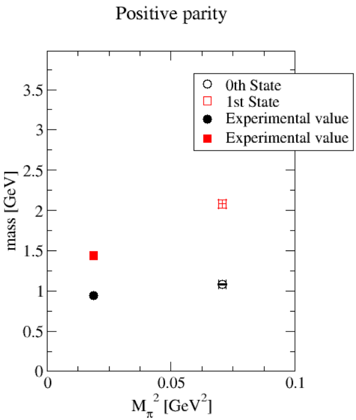


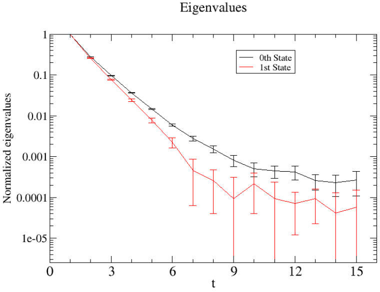
Nucleon ground state

1.068(6) GeV

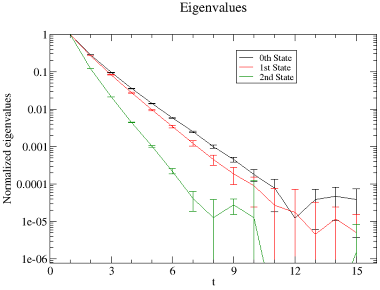
The negative parity sector

$N \to N : N^-$

280 configs, 6 interpolators: 32, 64 source and sink eigenvectors



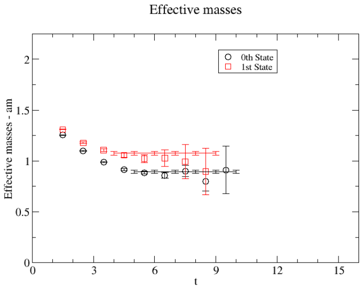
Traditional smeared sources

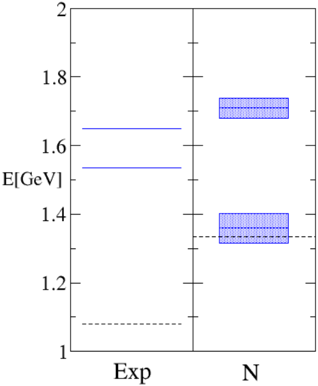


Distillation method

$N \to N : N^-$

280 configs, 6 interpolators: $32, 64$ source and sink eigenvectors





$N\pi$ scattering

A good interpolator for the pion-nucleon system is

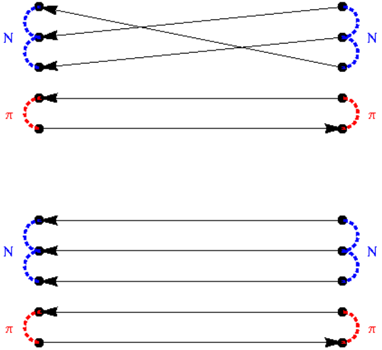
$$p(\mathbf{0}) = \sum_x P_\pm \epsilon_{abc} \, \Gamma_1 \, u_a(x) u_b^T(x) \Gamma_2 \, d_c(x) \qquad \pi^+(\mathbf{0}) = \sum_x \bar{d}(x) \gamma_5 \, u(x)$$

$$\boxed{N\pi(\mathbf{p} = \mathbf{0}) = \gamma_5 \, N(\mathbf{0}) \, \pi(\mathbf{0})}$$

An isospin projection is needed in order to overlap with the nucleon states $1/2^\pm$:

$$\boxed{O_{N\pi} = p\pi_0 + \sqrt{2} \, n\pi_+}$$

Non interacting $N\pi$ in S-wave



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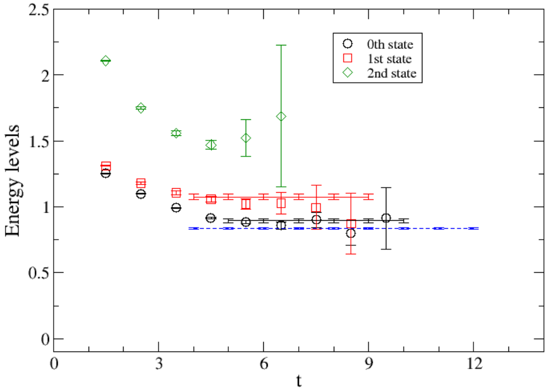
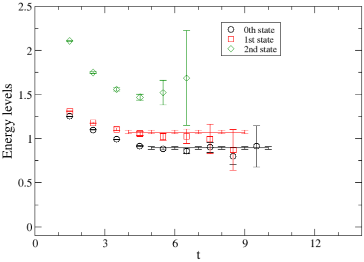
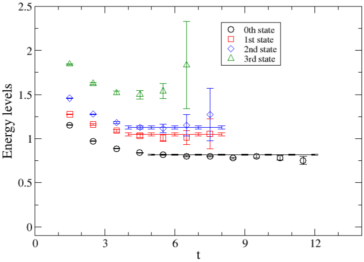


$N\pi$ in S-wave

280 configs, 7 interpolators: $32, 64$ source and sink eigenvectors

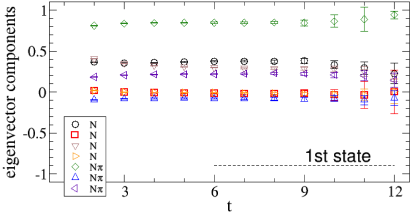


$N \to N$



$N\pi \to N\pi$

$N\pi$ in S-wave


eigenvector components
1
0.5
0
-0.5
-1
3
6
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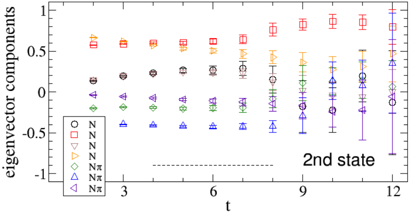
12
t
N
N
N
N
Nπ
Nπ
Nπ
1st state


eigenvector components
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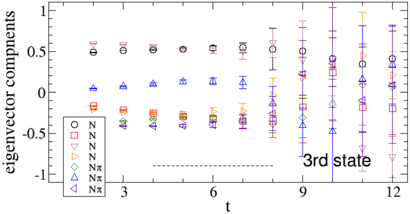
Nπ
Nπ
2nd state


eigenvector components
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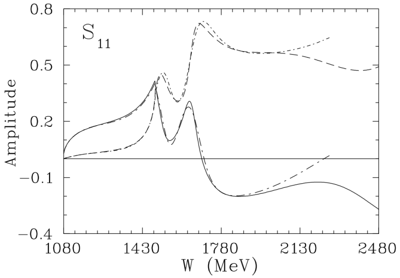
-0.5
-1
3
6
9
12
t
N
N
N
N
Nπ
Nπ
Nπ
3rd state

Energy levels: interpretation

For a system of interacting $N\pi$, the energy levels can be computed inverting Luescher relation

$$\tan\delta(q) = \frac{\pi^{3/2} q}{Z_{00}(1, q^2)} \qquad \textit{for} \qquad \mathbf{P} = \mathbf{0}$$

but a phase shift parametrization has to be assumed.



[Arndt et al., Phys.Rev.C74, 045205(2006)]

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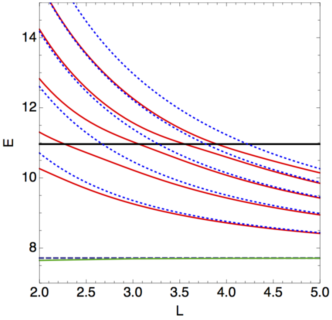

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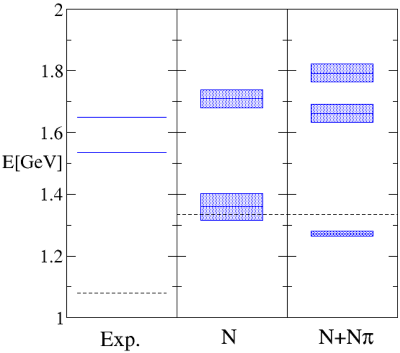

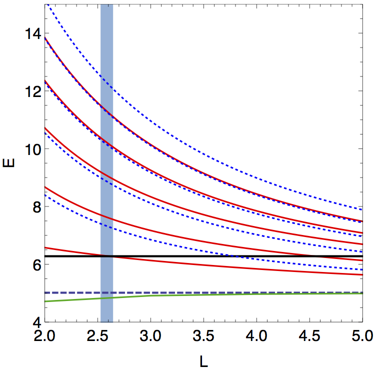

Resonace parameters

The phase shift profile can be fitted against the Breit-Wigner form in the vicinity of a resonance to evaluate the mass and width of the resonance.

$$\rho(s) = \sqrt{s}\,\Gamma(s)\cot\delta(s) = m_R^2 - s$$

$$m_R = 1.678(99)\ \mathrm{GeV}$$

For a different approach see [M. Doering et al., arXiv:1302.4065]

Summary

- Excited states still represent an outstanding challenge for lattice QCD.
- Two-particle system and phase shift analysis provide new information on the resonances of the QCD spectrum.
- A lot more has to be done!


UNI
GRAZ
Thank you!